

BRIEN MCMAHON HIGH SCHOOL

MATH DEPARTMENT

GEOMETRY B

SUMMER PACKET

2011

The problems in this packet are designed to help you review topics from previous mathematics courses that are important to your success in Geometry B. The topics covered in this packet should be mastered before entering Geometry. If topics have not been mastered, examples have been provided in each section.

Topics in this packet will be assessed within the first week of school through a test. Completion of this packet will help the student's 1st quarter grade.

While it is not required, it is recommended that students buy a calculator for their personal use throughout the school year. A scientific calculator will be sufficient for Geometry, however a TI – 83 graphing calculator is recommended for Algebra 2 and beyond.

Evaluate each Expression

Adding and Subtracting Integers:

$8 + (-2)$ is equal to $8 - 2$

If the terms have the *same* sign, *add* and keep that same sign.

$$2 + 3 = 5. \quad -2 + (-3) = -5. \quad -2 - 3 = -5.$$

If the terms have *opposite* signs, *subtract* the smaller from the larger, and keep the sign of the larger.

$$2 + (-3) = -1. \quad -2 + 3 = 1.$$

Any problems that look like this:

$$a - (-b)$$

Rewrite so that it looks like this:

$$a + b.$$

$2 - (-5)$ Two take away a negative 5 is $2 + 5$.

KEEP-CHANGE-CHANGE

$$-4 - (-7)$$

$$-4 + (+7)$$

Order of Operations

The rules for Order of Operations are as follows:

FIRST: Perform operations inside grouping symbols. Grouping symbols include parentheses (x) , brackets $[x]$, braces $\{x\}$, radical symbols \sqrt{x} , absolute value symbols $|x|$ and fraction bars $\frac{x}{y}$. If an expression contains more than one set of grouping symbols, simplify the expression inside the innermost set first. Follow the order of operations within that set of grouping symbols and then work outward.

SECOND: Simplify exponents.

THIRD: Perform multiplication and division from left to right. (Remember that a fraction indicates division.)

FOURTH: Perform addition and subtraction from left to right.

Hint: You can use the well-known phrase "Please Excuse My Dear Aunt Sally" to help you remember the Order of Operations. (Remember, however, that multiplication and division must be done in the order that they appear if they do not appear in parentheses. This is also true for addition and subtraction.)

Please Excuse My Dear Aunt Sally
Parentheses Exponents Multiplication Division Addition Subtraction

Examples for Topic IV:
Simplify.

A. $-4^2 + 24 \div 3 \cdot 2$	There are no (), work with exponent first.
$-16 + 24 \div 3 \cdot 2$	Note that there are no grouping symbols. Therefore the exponent only applies to the "4" and not "16".
$-16 + 8 \cdot 2$	Perform division because it appears first in the problem.
$-16 + 16$	Perform multiplication before addition
0	Perform addition
B. $4(25 - (5 - 2)^2)$	
$4(25 - (3)^2)$	Work with inner most parentheses first.
$4(25 - 9)$	Subtract inside inner most parentheses
$4(16)$	Continue with inner most parentheses, working with exponents.
64	Continue with inner most parentheses, subtract.
$3x - 4(8x + 2) + 5x$	Perform multiplication.
C. $3x - 4(8x + 2) + 5x$	
$3x - 32x - 8 + 5x$	Looking at the inner most parentheses, there are unlike terms and they cannot be simplified.
$-24x - 8$	Distribute the 4 to $(8x + 2)$.
	Combine like terms.

Solving Linear Equations

Follow the guideline below to help solve equations. The equation is solved when x is isolated. Check your answer.

FIRST: distribute if applicable to remove parentheses

SECOND: combine like terms of variables and constants on the **same** side of the equation

THIRD: if there are variables on both sides of the equation, bring them to one side by adding or subtracting

FOURTH: undo addition or subtraction

FIFTH: undo division or multiplication

(Remember, $\frac{x}{5}$ means "x divided by 5. To clear a fraction, multiply by its reciprocal).

Examples for Topic VI:

Solve for x.

A. $-2x + 7 = 15$	Subtract 7 from both sides of the equation.
$-2x = 8$	Divide both sides by -2
$x = -4$	X is isolated.
$-2(-4) + 7 = 15$	Check your answer by substituting your answer for x.
$15 = 15$	The solution is correct because both sides of the equation are equal.
B.	
$\frac{4}{5}x - 2 = 14$	Add 2 to both sides.
$\frac{4}{5}x = 16$	Multiply both sides of the equation by the reciprocal of the fraction $\left(\frac{5}{4}\right)$
$x = 20$	X is isolated.
$\frac{4}{5}(20) - 2 = 14$	Check your answer by substituting your answer for x.
$14 = 14$	The solution is correct because both sides of the equation are equal.
C.	
$-3(2x + 5) + 6x = 11x + 7$	Distribute.
$-6x - 15 + 6x = 11x + 7$	Combine like terms.
$-15 = 11x + 7$	Subtract 7 from both sides.
$-22 = 11x$	Divide both sides by -2
$-2 = x$	X is isolated.
$-3(2(-2) + 5) + 6(-2) = 11(-2) + 7$	Check your answer by substituting your answer for x.
$-15 = -15$	The solution is correct because both sides of the equation are equal.

Solving Proportions

$\frac{2}{x} = \frac{3}{9}$	Cross Multiply
$(9)(2) = (3)(x)$	Multiply 9 (2).
$18 = 3x$	Solve for x.
$\frac{18}{3} = \frac{3}{3}x$	Divide both sides by 3
$6 = x$	X is isolated.
$\frac{2}{6} = \frac{3}{9}$	These are equivalent fractions. The proportion is correct.

Exponents and Their Properties

Parts of Exponents: the large number is the base, the small number is the index, power or exponent.

Hint: Don't forget to follow the order of operations when simplifying

Index, or Power ↙
Base ↘
 10^2

Properties of Exponents

Multiplication: $(x^m)(x^n) = x^{(m+n)}$	Add exponents of terms with like bases
Division: $\frac{x^m}{x^n} = x^{(m-n)}$	Divide exponents of terms with like bases
Powers: $(x^m)^n = x^{(m \cdot n)}$	Multiply exponents
Zero Power: $x^0 = 1; x \neq 0$	Anything to the zero power is 1
Negative Exponents: $x^{-m} = \frac{1}{x^m}$	If a term has a negative exponent, change it's position in the fraction and the exponent becomes a positive
Fractions: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$	Apply the exponent to the numerator and denominator of a fraction

Graphing Linear Equation

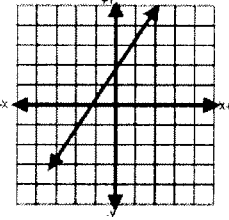
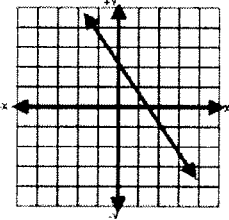
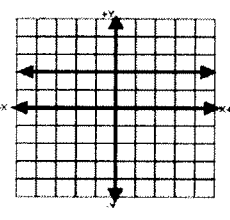
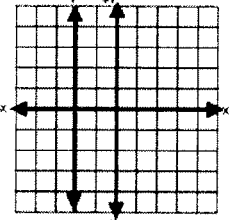
slope: $\frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$

Finding the slope given the graph of a line: Choose two points that are on the line. Count the vertical distance between the points. (Remember if you count "up" the number is positive and if you count "down" the number is negative.) Count the horizontal distance between the points. (Remember if you count "right" the number is positive and if you count "left" the number is negative.) Put the two numbers into a fraction with the vertical number in the numerator and the horizontal number in the denominator.

Equations of Lines: There are three main types of linear equations that are used in Algebra.

Name of Equation	Slope-Intercept Form	Point-Slope Form	Standard Form
Equation	$y = mx + b$	$y - y_1 = m(x - x_1)$	$Ax + By = C$
Variables	m = slope b = y-intercept (x, y) = any point on the line	m = slope (x_1, y_1) = any point on the line	A, B, C are coefficients (x, y) = any point on the line

Slope: Slope is the "m" in the slope-intercept equation. It represents the rate of change in the line and is often referred to as "rise over run". The four different types of slope are shown below.

Type of slope	Positive	Negative	Zero	Undefined
Graph				
Examples of Slopes	$m = \frac{2}{5}$ or $m = 3$	$m = \frac{-3}{4}$ or $m = -1$	$m = \frac{0}{2}$ or $m = 0$	$m = \frac{4}{0}$ or $m = \emptyset$
Examples of Equations	$y = 3x - 5$ $y = \frac{3}{4}x$	$y = -x + 2$ $y = -\frac{1}{5}x - 3$	$y = 3$ $y = -5$	$x = -4$ $x = 9$

Intercepts:

The x-intercept is the point on the graph where the line intersects the x-axis. At this point $y = 0$.

The y-intercept is the point on the graph where the line intersects the y-axis. At this point $x = 0$.

Graphing in slope-intercept form:

1. Write the equation in slope-intercept form
2. Plot the y-intercept (b) on the graph.
3. Turn the slope into a fraction if it is not already.
4. From the y-intercept, use the slope to plot three or more points on the graph and connect points to form a line.

Graphing by intercepts: Can be used with any form of an equation, but is usually associated with standard form

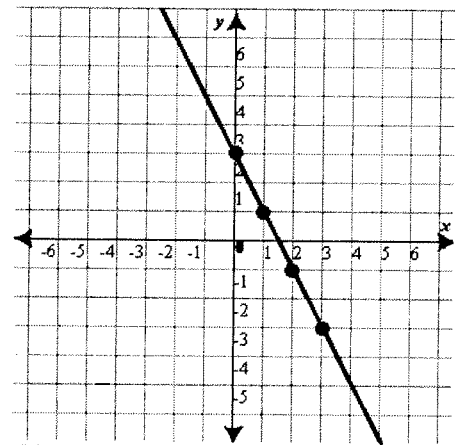
1. Find the x-intercept by making $y = 0$ and solving. Plot point (#, 0) on graph.
2. Find the y-intercept by making $x = 0$ and solving. Plot point (0, #) on graph.
3. Connect points to form line.

Examples for Topic VIII:

Graph the line.

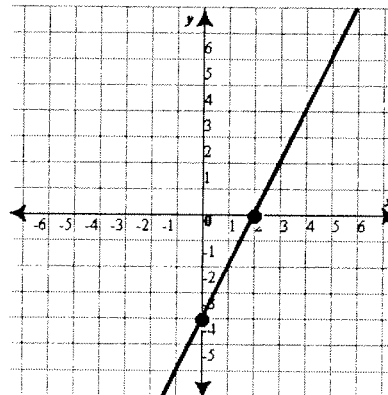
A. $y = -2x + 3$

1. Plot 0,3 because 3 is the y-intercept
2. Turn -2 into a fraction $-2 = \frac{-2}{1}$
3. Start at the y-intercept. Move down 2 and right 1, put a point. Repeat.
4. Connect points to create line.



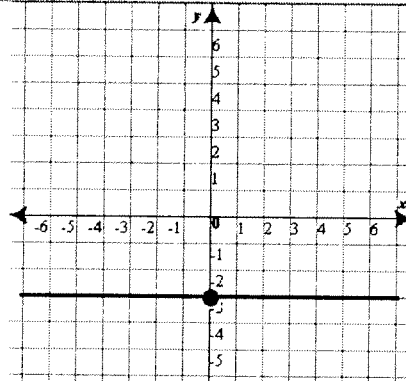
B. $6x - 3y = 12$

1. Calculate the x - intercept by making $y = 0$.
The x-intercept is (2, 0). Plot on graph.
2. Calculate the y - intercept by making $x = 0$.
The y-intercept is (0, -4). Plot on graph.
3. Connect points to create line.



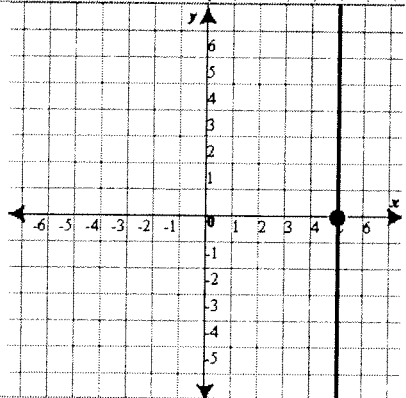
C. $y = -3$

1. The slope of this line is 0
2. Put a point on $y = -3$
3. Draw a horizontal line.



D. $x = 5$

1. The slope of this line is \emptyset
2. Put a point on $x = 5$
3. Draw a vertical line.



Determining Linear Equations from Graphs

y-intercept: Look for the point where the line intersects the y-axis.

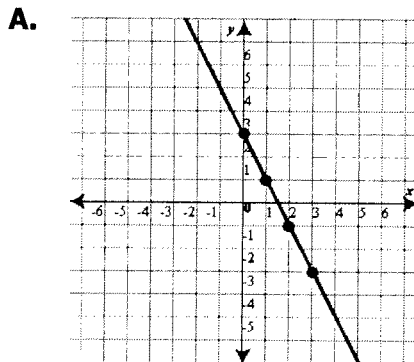
x-intercept: Look for the point where the line intersects the x-axis.

slope: Choose two points that are on the line. Count the vertical distance between the points. (Remember if you count "up" the number is positive and if you count "down" the number is negative.) Count the horizontal distance between the points. (Remember if you count "right" the number is positive and if you count "left" the number is negative.) Put the two numbers into a fraction with the vertical number in the numerator and the horizontal number in the denominator.

Equation: To find the equation of a line from a graph, locate the y-intercept and determine the slope. Write the equation in slope-intercept form.

Examples for Topic IX:

Find the slope, y-intercept, x-intercept and the slope-intercept equation of each line.



y-intercept: (0,3)

x-intercept: (1.5, 0) – estimated

slope:

- Choose two points (0,3) and (1,1)
- To move from the first to the second, count down 2 and right 1.
- Write as a fraction and simplify:

$$\frac{-2}{1} = -2$$

Equation: $y = -2x + 3$

(x-intercept not needed to write the equation)

Working Without Graphs

Equations Needed:

Name of Equation	Slope-Intercept Form	Point-Slope Form	Slope Formula
Equation	$y = mx + b$	$y - y_1 = m(x - x_1)$	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Variables	m = slope b = y-intercept (x, y) = any point on the line	m = slope (x_1, y_1) = any point on the line	m = slope (x_1, y_1) = a point on the line (x_2, y_2) = a 2 nd point on the line
When to Use	With the slope and y-intercept	With the slope and a point	With two points

Perpendicular and Parallel Lines:

- **Parallel Lines:** parallel lines have the same slope
- **Perpendicular Lines:** perpendicular lines have slopes that are opposite reciprocals of each other

Name of Equation	Parallel Lines	Perpendicular Lines
Equation #1	$y = m_1x + b$	$y = m_1x + b$
Equation #2	$y = m_2x + b$	$y = -\frac{1}{m_1}x + b$
Slopes	$m_1 = m_2$	$m_1 \cdot \left(-\frac{1}{m_1}\right) = -1$

Intercepts:

- **y-intercept:** to calculate the y-intercept, replace x with 0 and solve
- **x-intercept:** to calculate the x-intercept, replace y with 0 and solve

Examples for Topic X:

- a. Find the equation of each line. Leave your answer in slope-intercept form.

A. $m=3$; passing through $(2,-1)$	Use point slope formula because you are given a point and the slope
$y+1=2(x-2)$	Substitute the 3 for the m , and $(2,-1)$ for (x_1, y_1)
$y+1=2x-4$	Distribute.
$y=2x-5$	Solve for y .
B. Passing through $(2,-3)$ and $(5, 6)$	Use the slope formula to calculate slope.
$\frac{\text{Change}}{\text{Original}} = \frac{x}{100}$	Substitute $(2,-1)$ for (x_1, y_1) and $(5, 6)$ for (x_2, y_2) . Simplify.
$y-6=3(x-5)$	Use point slope formula with the slope you calculated and one of the points from the problem
$y-6=3x-15$	Distribute.
$y=3x-9$	Solve for y .

Percents

To find the percent of a number you can set it up as a proportion. Percents are out of 100.

$$\frac{20 - \text{is}}{100 - \text{of}} = \frac{x - \text{is}}{50 - \text{of}}$$

Percent Change: The ratio of amount of change to the original amount expressed as a percent:

$$\frac{\text{Change}}{\text{Original}} = \frac{x}{100}$$

Example: The price of sweater decreased from \$29.99 to \$24.49. Find the percent of decrease:

	Set up the proportion
$\frac{5.50}{29.99} = \frac{x}{100}$	
$550 = 29.99x$	Cross Multiply
$18 = x$	Solve for x

Scientific Notation

Scientific Notation is based on powers of the base number 10.

The number 123,000,000,000 in scientific notation is written as :

$$1.23 \times 10^{11}$$

The first number 1.23 is called the coefficient. It must be greater than or equal to 1 and less than 10.

The second number is called the base . It must always be 10 in scientific notation. The base number 10 is always written in exponent form. In the number 1.23×10^{11} the number 11 is referred to as the exponent or power of ten.

Writing numbers in scientific Notation:	
3,000	3×10^3
Writing numbers in standard notation:	
8.25×10^5 becomes: 825,000	You move the decimal five places to the right (positive 5)
5.2×10^{-4} becomes: .00052	You move the decimal four places to the left. (negative you go to the left)

Word Problems

Apply information from previous problems to solve real-world problems.

Problem Situation: Beth and Paul each improved their yards by planting rose bushes and geraniums. They bought their supplies from the same store. Beth spent \$130 on 5 rose bushes and 11 geraniums. Paul spent \$70 on 10 rose bushes and 3 geraniums. What is the cost of one rose bush and the cost of one geranium.

A. Assign variables. Let r = rose bush
and g = geranium.

$$\begin{aligned} 5r + 11g &= 130 \text{ - Beth's} \\ 10r + 3g &= 70 \end{aligned}$$

Write the 1st equation.

Write the 2nd equation.

$$2 \bullet ((5r + 11g) = 130)$$

$$5r + 11g = 130$$

$$5(4) + 11(10) = 130$$

$$0r + 19g = 190$$

In order to combine to eliminate- you have to multiply the first equation by 2. Now we can subtract to eliminate the r variable.

Solve for g .

$$g = \frac{190}{19}$$

$g = 10$, Now plug in for g to find r .

$$10r + 3g = 70$$

$$10r + 3(10) = 70$$

$$10r + 30 = 70$$

$$10r + 30 - (30) = 70 - (30)$$

$$10r = 40$$

$$r = \frac{40}{10}$$

$$r = 4$$

Use the original equation to Check your work:

$$5r + 11g = 130$$

$$5(4) + 11(10) = 130$$

Example of a tax problem:

Ricardo and Jonathan go to Burger Barn for lunch. They order two cheeseburgers for \$2.50 each and a large fry for \$3.25. If sales tax is 5.5%, calculate the final cost of their lunch.

Step 1: Add up the amount :

$2 \times \$2.50$ (2 cheeseburgers) = \$5.00 + 3.25 (1 large fry) = \$8.25 (Cost of food minus the tax)

Now you have to find 5.5% of \$8.25. There are two ways to do this:

1. Set it up as a proportion:

$$\frac{5.5}{100} = \frac{x}{8.25}$$

$$100x = 45.375$$

$$x = \frac{45.375}{100}$$

$$x = .45375$$

The tax is \$0.45. Add to the pretax price: $8.25 + 0.45 = \$8.70$

Second way-

Convert 5.5% into a decimal: $.055 = 5.5\%$. Multiply to find 5.5% of \$8.25

$$(\$8.25) (.055) = \$0.46.$$

Now add the tax to the pretax cost: $\$8.25 + \$0.46 = \$ 8.71$

Plotting Points on the Coordinate Plane

Suppose you were told to locate "(5, 2)" where would you look?

To understand the meaning of "(5, 2)", you have to know the following rule: The x -coordinate (the number for the x -axis) *always* comes first. The first number (the first coordinate) is *always* on the horizontal axis. Remember, for positive x -values move right, for negative x -values move left; and for positive y -values move up, for negative y -values move down.

