

I. Mental Math

Adding from left to right: Add the largest place values first and then work your way right, to the ones column.

Multiplying from left to right: Multiply the left most column and work your way to the ones column. Add the results.

Examples for Topic I:

Simplify the following problems using mental math (NO CALCULATOR).

A. $79+67$

$$70+60=130$$

Add the tens column.

$$9+7=16$$

Add the ones column.

$$130+16=146$$

Add the results.

B. $124 \cdot 3$

$$100 \cdot 3 = 300$$

Multiply the hundreds column by 3.

$$20 \cdot 3 = 60$$

Multiply the tens column by 3

$$4 \cdot 3 = 12$$

Multiply the ones column by 3.

$$300+60+12=372$$

Add the results.

Practice Problems for Topic I:

Simplify the following problems using mental math (NO CALCULATOR).

a) $325+251$

b) $85 \cdot 6$

c) $3952 \cdot 2$

Answers: a) $500+70+6=576$ b) $480+30=510$ c) $6000+1800+100+4=7904$

II. Prime Factorization

Prime Numbers: Prime numbers are numbers that are only divisible by 1 and themselves. For example, the only numbers that divide evenly into 7 are 1 and 7. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.....

Factors: A factor is a term that is part of another term when multiplied, or the numbers that are multiplied together to get another number.

Example: What are the factors of 15?

$$5 \cdot 3 = 15 \text{ (3 and 5 are the factors of 15)}$$

Prime Factorization: The prime numbers that are multiplied together to make the original number.

Example: What is the prime factorization of 20?

$$20 = 2 \cdot 2 \cdot 5 = 2^2 \cdot 5$$

Examples for Topic II:

Find the prime factorization for each number.

A. 18	Start with the smallest prime number, 2 and check to see if 2 can divide 18.
2(9)	Working with the 9, check to see if 2 can divide 9. It cannot, so check if 3 (the next prime number) can divide 9.
2·3·3	It is not possible to divide further because 2 and 3 are both prime numbers.
2·3 ²	Write the final answer using exponents.

B. 350	Start with the smallest prime number, 2 and check to see if 2 can divide 350.
2(175)	Working with the 175, check to see if 2 can divide 175. It cannot, so check if the next prime number (3 or 5, etc.) can divide 175.
2·5(35)	Working with the 35, check to see which of the next prime numbers can divide 35.
2·5·5(7)	It is not possible to divide further because 2, 5 and 7 are both prime numbers.
2·5 ² ·7	Write the final answer using exponents.

Practice Problems for Topic II:

Write the prime factorization for the numbers below.

a) 80

b) 156

c) 1365

Answers: a) $2^4 \cdot 5$ b) $2^2 \cdot 3 \cdot 13$ c) $3 \cdot 5 \cdot 7 \cdot 13$

III: Real Numbers and Their Properties

Number Systems:

- **Natural Number** (Counting Numbers) 1, 2, 3, ...
- **Whole Numbers** (Introduce Zero) 0, 1, 2, 3,
- **Integers** (Introduce Negative Numbers) ..., -3, -2, -1, 0, 1, 2, 3,
- **Rational Numbers** (Numbers that can be written as the ratio of two integers.) When written as decimals, Rational Numbers terminate or repeat.
Examples: $\frac{2}{3} = 0.666\dots$ $\frac{3}{4} = 0.75$ $\frac{-7}{1} = -7$
Note: The above number systems do not contain fractions or decimals.
- **Irrational Numbers** - Number that when written as decimals neither terminate nor repeat.
Examples: $\pi = 3.1415\dots$ $\sqrt{5} = 2.236067978$
- **Real Numbers** (All numbers on the number line.) This system combines the Natural #s, Whole #s, Integers, Rational #s, and Irrational #s.

Properties of Real Numbers: Let m , n , and p be positive real numbers.

Property	Addition	Multiplication
COMMUTATIVE	$m + n = n + m$	$mn = nm$
ASSOCIATIVE	$(m + n) + p = m + (n + p)$	$(mn)p = m(np)$
IDENTITY	$m + 0 = m, 0 + m = m$	$m \times (1) = m, 1 \times (m) = m$
INVERSE	$m + (-m) = 0$	$m \times \left(\frac{1}{m}\right) = 1, m \neq 0$
DISTRIBUTIVE	$m(n + p) = mn + mp$	

- **Additive inverse:** the opposite of any $m = -m$
 Examples: the additive inverse of 4 is -4 the additive inverse of -3 is 3
- **Multiplicative inverse:** the reciprocal; the inverse of m is $\frac{1}{m}$
 Examples: the multiplicative inverse of $\frac{1}{3} = 3$ the multiplicative inverse of $\frac{5}{6} = \frac{6}{5}$

Practice Problems for Topic III:

a. Classify the following numbers

a) -12

b) $\sqrt{19}$

c) $\frac{3}{7}$

b. Identify the property being performed.

d) $(3)(8) = (8)(3)$

e) $-6 + 0 = -6$

f) $3(4 + 5) = 3(4) + 3(5)$

Answers: a) Integer, Rational Number, Real Number b) Irrational Number, Real Number c) Rational Number, Real Number d) Associative Property of Multiplication e) Additive Identify f) Distributive Property

IV. Order of Operations

The rules for Order of Operations are as follows:

FIRST: Perform operations inside grouping symbols. Grouping symbols include parentheses (x) , brackets $[x]$, braces $\{x\}$, radical symbols \sqrt{x} , absolute value symbols $|x|$ and fraction bars $\frac{x}{y}$. If an expression contains more than one set of grouping symbols, simplify the expression inside the innermost set first. Follow the order of operations within that set of grouping symbols and then work outward.

SECOND: Simplify exponents.

THIRD: Perform multiplication and division from left to right. (Remember that a fraction indicates division.)

FOURTH: Perform addition and subtraction from left to right.

Hint: You can use the well-known phrase "Please Excuse My Dear Aunt Sally" to help you remember the Order of Operations. (Remember, however, that multiplication and division must be done in the order that they appear if they do not appear in parentheses. This is also true for addition and subtraction.)

Please Excuse My Dear Aunt Sally
Parentheses Exponents Multiplication Division Addition Subtraction

Examples for Topic IV:
Simplify.

A. $-4^2 + 24 \div 3 \cdot 2$	There are no (), work with exponent first.
$-16 + 24 \div 3 \cdot 2$	Note that there are no grouping symbols. Therefore the exponent only applies to the "4" and not the "-".
$-16 + 8 \cdot 2$	Perform division because it appears first in the problem.
$-16 + 16$	Perform multiplication before addition
0	Perform addition

B. $4(25 - (5 - 2)^2)$	Work with inner most parentheses first.
$4(25 - (3)^2)$	Subtract inside inner most parentheses
$4(25 - 9)$	Continue with inner most parentheses, working with exponents.
$4(16)$	Continue with inner most parentheses, subtract.
64	Perform multiplication.

C. $3x - 4(8x + 2) + 5x$	Looking at the inner most parentheses, there are unlike terms and they cannot be simplified.
$3x - 32x - 8 + 5x$	Distribute the 4 to (8x+2).
$-24x - 8$	Combine like terms.

Practice Problems for Topic IV:
Simplify.

a) $\frac{5^2 - 1}{8 - (2)(3)}$

b) $3(-4x + 1) - 2(x - 2)$

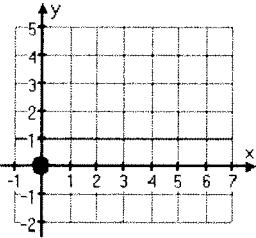
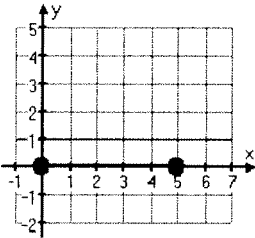
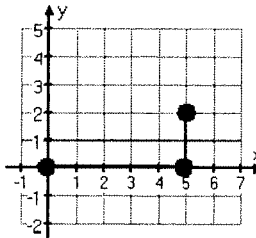
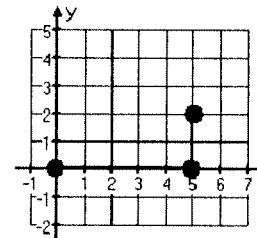
c) $-5^2 + 30 \div 5(4)$

Answers: a) $\frac{24}{2} = 12$ b) $-12x + 3 - 2x + 4 = -14x + 7$ c) $-25 + 6 \cdot 4 = -1$

V. Plotting Points on the Coordinate Plane

Suppose you were told to locate "(5, 2)" where would you look?

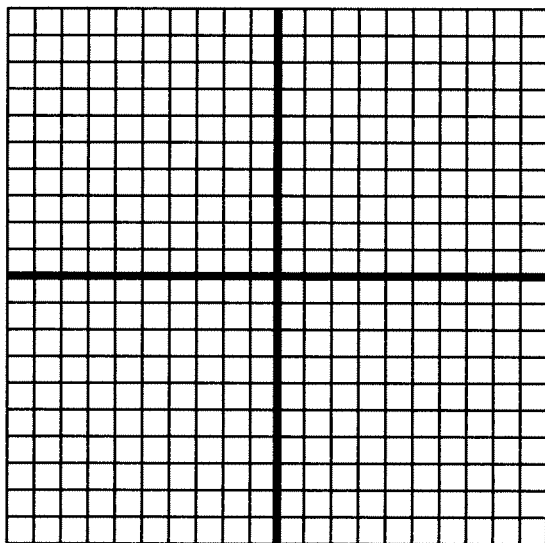
To understand the meaning of "(5, 2)", you have to know the following rule: The x -coordinate (the number for the x -axis) *always* comes first. The first number (the first coordinate) is *always* on the horizontal axis. Remember, for positive x -values move right, for negative x -values move left; and for positive y -values move up, for negative y -values move down.

<p>For point (5,2), begin at the origin, the spot where the axes cross...</p> 	<p>...count right five to 5 on the x-axis...</p> 	<p>...then count up two to the point corresponding to 2 on the y-axis...</p> 	<p>...then put the point on the graph</p> 
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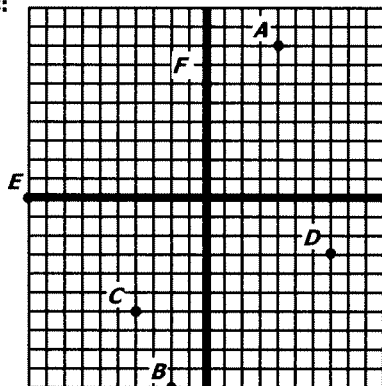
Practice Problems for Topic V.

Plot the points on the graph and label them.

- | | | |
|--------------|---------------|---------------|
| a) $A(4,8)$ | b) $B(-2,10)$ | c) $C(-4,-6)$ |
| d) $D(7,-3)$ | e) $E(-10,0)$ | f) $F(0,6)$ |



Answers:



VI: Solving Linear Equations

Follow the guideline below to help solve equations. The equation is solved when x is isolated. Check answer.

FIRST: distribute if applicable to remove parentheses

SECOND: combine like terms of variables and constants on the **same** side of the equation

THIRD: if there are variables on both sides of the equation, bring them to one side by adding or subtracting

FOURTH: undo addition or subtraction

FIFTH: undo division or multiplication

(Remember, $\frac{x}{5}$ means "x divided by 5. To clear a fraction, multiply by its reciprocal).

Examples for Topic VI:

Solve for x .

A. $-2x + 7 = 15$	Subtract 7 from both sides of the equation.
$-2x = 8$	Divide both sides by -2
$x = -4$	x is isolated.
$-2(-4) + 7 = 15$	Check your answer by substituting your answer for x .
$15 = 15$	The solution is correct because both sides of the equation are equal.

B. $\frac{4}{5}x - 2 = 14 + x$	Add 2 to both sides.
$\frac{4}{5}x = 16 + x$	Multiply both sides of the equation by the denominator of the fraction (5)
$4x = 80 + 5x$	Subtract $5x$ from both sides.
$-x = 80$	Divide both sides by -1, $x = -80$.
$\frac{4}{5}(-80) - 2 = 14 - 80$	Check your answer by substituting your answer for x .
$-66 = -66$	The solution is correct because both sides of the equation are equal.

C. $-3(2x + 5) + 6x = 11x + 7$	Distribute.
$-6x - 15 + 6x = 11x + 7$	Combine like terms.
$-15 = 11x + 7$	Subtract 7 from both sides.
$-22 = 11x$	Divide both sides by -2
$-2 = x$	x is isolated.
$-3(2(-2) + 5) + 6(-2) = 11(-2) + 7$	Check your answer by substituting your answer for x .
$-15 = -15$	The solution is correct because both sides of the equation are equal.

Practice Problems for Topic IV:

a. Simplify.

a) $3(x - 4) = 8 - x$

b) $\frac{2}{7}(x - 14) = 23 - x$

c) $5y \div 6 = 75$

Answers: a) $x = 5$ b) $x = 7$ c) $y = 90$

VII: Linear Equations Word Problems

When solving a word problem, the unknown is "x". Create an equation to model the situation and then solve.

Examples for Topic V:

Write an equation to help you solve the word problems below. Define the variable. Show all work.

- A. The length of a rectangle is 4 cm greater than its width. The perimeter of the rectangle is 25 find the dimensions of the rectangle.

$x = \text{width of rectangle}$ $l = x + 4$	Define the variable. Since the width of the rectangle is completely unknown, x represents the width. Since the width is "x", the length is "x+4" because it is 4 cm greater than the width.
$P = 2w + 2l$ $25 = 2x + 2(x + 4)$	Set-up an equation.
$25 = 4x + 8$ $x = 4.25$	Solve.
$w = 4.25 \text{ cm}$ $l = 4.25 + 4 = 8.25 \text{ cm}$	Use the answer to find the solution to the word problem.

- B. A new cellphone plan charges \$35 for unlimited calls each month, but charges \$0.08 for every text message sent. If Paul's bill was \$85.16, how many text messages did he send?

$x = \# \text{ of texts sent}$	Define the variable. Since the # of texts is unknown, x represents the # of texts.
$35 + 0.08x = 85.16$	Set-up an equation. \$35 is the starting price of the plan. "x" is multiplied by 0.08 because the charge is \$0.08 for every text. The total cost of the plan for the month is \$85.16.
$x = 627$	Solve.
Paul sent 627 texts.	Use the answer to find the solution to the word problem.

- C. The sum of 3 consecutive integers is 339. Find the numbers.

$x = 1^{\text{st}} \text{ integer}$ $(x + 1) = 2^{\text{nd}} \text{ integer}$ $(x + 2) = 3^{\text{rd}} \text{ integer}$	Define the variable. Since the first integer is completely unknown, x represents the first integer. The value of the second integer is one more than the first, so the second integer is "x + 1". The value of the third integer is two more than the first, so the third integer is "x + 2".
$x + (x + 1) + (x + 2) = 339$	Set-up an equation. Add all integers together and total them to 339.
$3x + 3 = 339$ $x = 112$	Solve.
$1^{\text{st}} \text{ integer} = x = 112$ $2^{\text{nd}} \text{ integer} = x + 1 = 113$ $3^{\text{rd}} \text{ integer} = x + 2 = 114$	Use the answer to find the solution to the word problem.

Practice Problems for Topic VII:

Write an equation to help you solve the word problems below. Define the variable. Show all work.

- a) The sum of four consecutive integers is 266. Find the integers.
- b) Joe has budgeted \$1800 to spend on chairs for his new restaurant. He found a company that charges \$70 per chair, but he has to pay \$610 for shipping. How many chairs can he purchase?

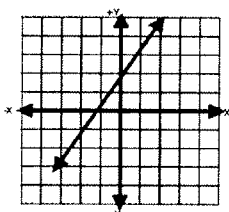
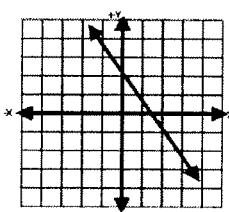
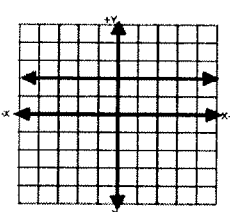
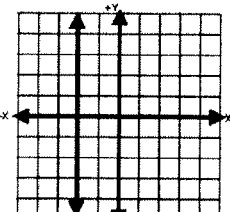
Answers: a) 65, 66, 67, 68 b) 17 chairs

VIII: Graphing Linear Equations

Equations of Lines: There are three main types of linear equations that are used in Algebra.

Name of Equation	Slope-Intercept Form	Point-Slope Form	Standard Form
Equation	$y = mx + b$	$y - y_1 = m(x - x_1)$	$Ax + By = C$
Variables	$m = \text{slope}$ $b = \text{y-intercept}$ $(x, y) = \text{any point on the line}$	$m = \text{slope}$ $(x_1, y_1) = \text{any point on the line}$	A, B, C are coefficients $(x, y) = \text{any point on the line}$

Slope: Slope is the "m" in the slope-intercept equation. It represents the rate of change in the line and is often referred to as "rise over run". The four different types of slope are shown below.

Type of slope	Positive	Negative	Zero	Undefined
Graph				
Examples of Slopes	$m = \frac{2}{5}$ or $m = 3$	$m = -\frac{3}{4}$ or $m = -1$	$m = \frac{0}{2}$ or $m = 0$	$m = \frac{4}{0}$ or $m = \emptyset$
Examples of Equations	$y = 3x - 5$ $y = \frac{3}{4}x$	$y = -x + 2$ $y = -\frac{1}{5}x - 3$	$y = 3$ $y = -5$	$x = -4$ $x = 9$

Intercepts:

The x-intercept is the point on the graph where the line intersects the x-axis. At this point $y = 0$.
The y-intercept is the point on the graph where the line intersects the y-axis. At this point $x = 0$.

Graphing in slope-intercept form:

1. Write the equation in slope-intercept form
2. Plot the y-intercept (b) on the graph.
3. Turn the slope into a fraction if it is not already.
4. From the y-intercept, use the slope to plot three or more points on the graph and connect points to form a line.

Graphing by intercepts: Can be used with any form of an equation, but is usually associated with standard form

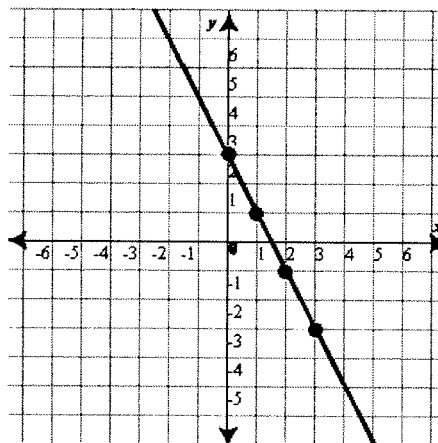
1. Find the x-intercept by making $y = 0$ and solving. Plot point (#, 0) on graph.
2. Find the y-intercept by making $x = 0$ and solving. Plot point (0, #) on graph.
3. Connect points to form line.

Examples for Topic VIII:

Graph the line.

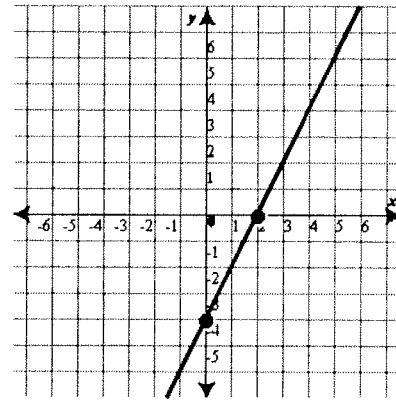
A. $y = -2x + 3$

1. Plot 0,3 because 3 is the y-intercept
2. Turn -2 into a fraction $-2 = -\frac{2}{1}$
3. Start at the y-intercept. Move down 2 and right 1, put a point. Repeat.
4. Connect points to create line.



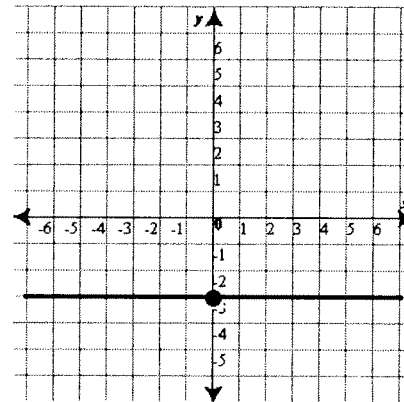
B. $6x - 3y = 12$

1. Calculate the x – intercept by making $y = 0$.
The x-intercept is $(2, 0)$. Plot on graph.
2. Calculate the y – intercept by making $x = 0$.
The y-intercept is $(0, -4)$. Plot on graph.
3. Connect points to create line.



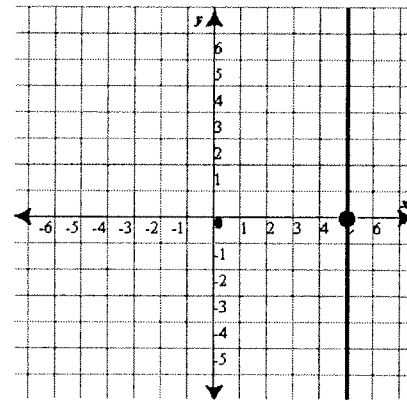
C. $y = -3$

1. The slope of this line is 0
2. Put a point on $y = -3$
3. Draw a horizontal line.



D. $x = 5$

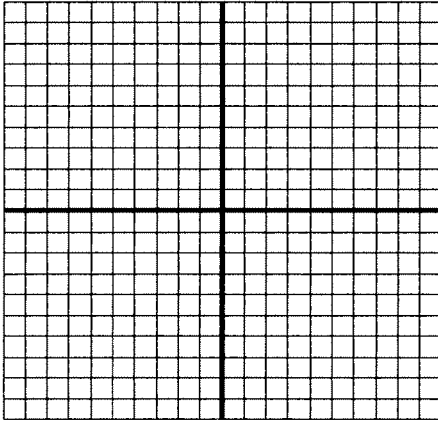
1. The slope of this line is \emptyset
2. Put a point on $x = 5$
3. Draw a vertical line.



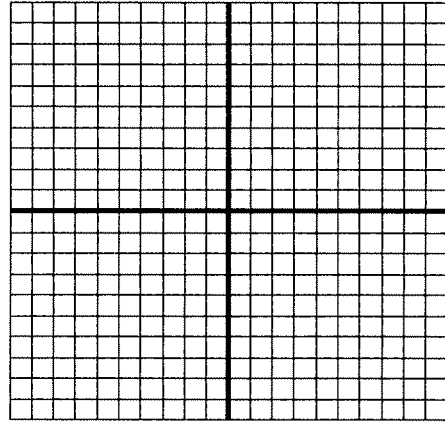
Practice Problems for Topic VIII:

Graph.

a) $y = \frac{3}{4}x - 4$

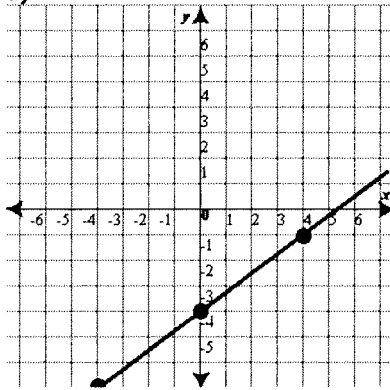


b) $5x - 4y = -20$

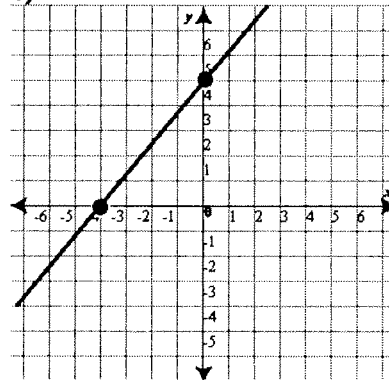


Answers:

a)



b)



IX: Determining Linear Equations from Graphs

y-intercept: Look for the point where the line intersects the y-axis.

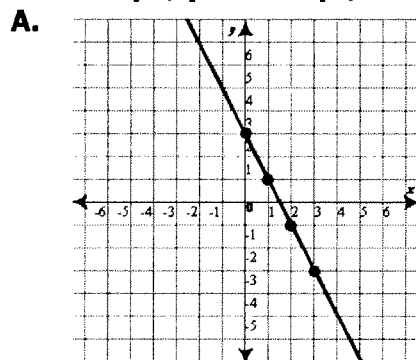
x-intercept: Look for the point where the line intersects the x-axis.

slope: Choose two points that are on the line. Count the vertical distance between the points. (Remember if you count "up" the number is positive and if you count "down" the number is negative.) Count the horizontal distance between the points. (Remember if you count "right" the number is positive and if you count "left" the number is negative.) Put the two numbers into a fraction with the vertical number in the numerator and the horizontal number in the denominator.

Equation: To find the equation of a line from a graph, locate the y-intercept and determine the slope. Write the equation in slope-intercept form.

Examples for Topic IX:

Find the slope, y-intercept, x-intercept and the slope-intercept equation of each line.



y-intercept: (0,3)

x-intercept: (1.5, 0) – estimated

slope:

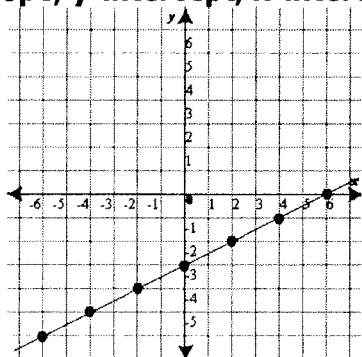
- Choose two points (0,3) and (1,1)
- To move from the first to the second, count down 2 and right 1.
- Write as a fraction and simplify: $\frac{-2}{1} = -2$

Equation: $y = -2x + 3$

(x-intercept not needed to write the equation)

Practice Problems for Topic IX:

Find the slope, y-intercept, x-intercept and the slope-intercept equation of each line.



a) y –intercept: _____

b) x – intercept: _____

c) slope: _____

d) equation: _____

Answers: a) (0,-3) b) (6,0) c) $\frac{1}{2}$ d) $y = \frac{1}{2}x - 3$

X: Working Without Graphs

Equations Needed:

Name of Equation	Slope-Intercept Form	Point-Slope Form	Slope Formula
Equation	$y = mx + b$	$y - y_1 = m(x - x_1)$	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Variables	m = slope b = y-intercept (x, y) = any point on the line	m = slope (x_1, y_1) = any point on the line	m = slope (x_1, y_1) = a point on the line (x_2, y_2) = a 2 nd point on the line
When to Use	With the slope and y-intercept	With the slope and a point	With two points

Perpendicular and Parallel Lines:

- **Parallel Lines:** parallel lines have the same slope
- **Perpendicular Lines:** perpendicular lines have slopes that are opposite reciprocals of each other

Name of Equation	Parallel Lines	Perpendicular Lines
Equation #1	$y = m_1x + b$	$y = m_1x + b$
Equation #2	$y = m_2x + b$	$y = -\frac{1}{m_1}x + b$

Slopes	$m_1 = m_2$	$m_1 \cdot \left(-\frac{1}{m_1}\right) = -1$
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Intercepts:

- **y-intercept:** to calculate the y-intercept, replace x with 0 and solve
- **x-intercept:** to calculate the x-intercept, replace y with 0 and solve

Examples for Topic X:

a. Find the equation of each line. Leave your answer in slope-intercept form.

A. $m=3$; passing through $(2,-1)$	Use point slope formula because you are given a point and the slope
$y+1=2(x-2)$	Substitute the 3 for the m , and $(2,-1)$ for (x_1, y_1)
$y+1=2x-4$	Distribute.
$y=2x-5$	Solve for y.

B. Passing through $(2,-3)$ and $(5, 6)$	Use the slope formula to calculate slope.
$m = \frac{6+3}{5-2} = \frac{9}{3} = 3$	Substitute $(2,-1)$ for (x_1, y_1) and $(5, 6)$ for (x_2, y_2) . Simplify.
$y-6=3(x-5)$	Use point slope formula with the slope you calculated and one of the points from the problem
$y-6=3x-15$	Distribute.
$y=3x-9$	Solve for y.

C. Vertical line passing through $(4, 2)$	Any vertical line has the equation $x = \#$
$x = 4$	The x-value is 4, so substitute 4 for the "#" sign

D. Perpendicular to $y=2x+4$ and passing through $(-6, 1)$	Use the slope of the given equation to find the slope of the new equation
$2 \rightarrow -\frac{1}{2}$	Perpendicular lines have slopes that are opposite reciprocals. (Take the 2, find the reciprocal and change the sign).
$y-1 = -\frac{1}{2}(x+6)$	Use point slope formula with the slope you found and the given point
$y-1 = -\frac{1}{2}x-3$	Distribute
$y = -\frac{1}{2}x-2$	Solve for y.

b. Find the x- and y-intercepts and the slope of the line.

E. $4x-3y=15$		
Slope: rearrange the equation to solve for y.	x-intercept: substitute 0 for y-value and solve.	y-intercept: substitute 0 for x-value and solve.
$-3y = -4x+15$	$4x-3(0) = 15$	$4(0)-3y = 15$
$y = \frac{4}{3}x-5$	$4x = 15$	$-3y = 15$
The slope is $\frac{4}{3}$	$x = \frac{15}{4} = 3.75$	$y = -5$
	The x-intercept is $(3.75, 0)$	The y-intercept is $(0, -5)$

Practice Problems for Topic X:

a. Find the equation of each line. Leave your answer in slope-intercept form.

a) $m = -3$ through point $(-5, 19)$

b) Horizontal line through point $(7, -2)$

c) Parallel to $x - 4y = 1$ through point $(8, 2)$

d) Through points $(-2, -4)$ and $(8, 1)$

b. Find the x- and y-intercepts and the slope of the line.

e) $6x - 3y = -12$

f) $y = \frac{1}{2}x + 5$

Slope: _____

Slope: _____

X – intercept: _____

X – intercept: _____

y- intercept: _____

y- intercept: _____

Answers: **a)** $y = -3x + 4$ **b)** $y = -2$ **c)** $y = \frac{1}{4}x$ **d)** $y = \frac{1}{2}x - 3$ **e)** slope: 2; x – intercept: $(-2, 0)$; y – intercept: $(0, 4)$ **f)** slope: $\frac{1}{2}$; x – intercept: $(-10, 0)$; y – intercept: $(0, 5)$

XI: Completing Tables

Tables have two columns, one for the x-value (the dependent variable) and one for the y-value (the independent variable).

Calculating y-value: take the x-value and substitute it in the equation for x. Simplify.

Calculating x-value: set the y-value equal to the equation and solve for x.

Examples for Topic XI:

Complete the table.

x	$-3x + 4$	
0	4	Substitute 0 for x and simplify. $-3(0) + 4 = 4$
-3	13	Substitute -3 for x and simplify. $-3(-3) + 4 = 13$
5	-11	Substitute 5 for x and simplify. $-3(5) + 4 = -11$
4	-8	Set $-3x + 4$ equal to -8 and solve. $-3x + 4 = -8$

-14	46
-----	----

Set $-3x + 4$ equal to 46 and solve. $-3x + 4 = 46$

Practice Problems for Topic XI:
Complete the table.

	x	$6x - 10$
a)	0	
b)	-4	
c)	11	
d)		-22
e)		38

	x	$x^2 - 1$
f)	0	
g)	-4	
h)	11	
i)	-3	
j)	20	

Answers: a) -10 b) -34 c) 56 d) -2 e) 8 f) -1 g) 15 h) 120 i) 8 j) 399

XII: Two Variable Word Problems

Apply information from problem sets VIII – XII to solving real-world problems.

Examples for Topic XII:

Problem Situation: Erin and Da'shia go on a road trip to Bakersville, which is 257 miles away. The two girls have already traveled 57 miles and are traveling at a rate of 60 miles per hour.

- A. Write an equation to model the distance, d , the girls travel in h hours.

Answer: $d = 60h + 57$

Let d = distance and h = hours. They have already traveled 57 miles and continue to travel 60 miles per hour.

- B. How far will they be in 3 hours?

Answer: $d = 60(3) + 57 = 180 + 57 = 237 \text{ miles}$

The given information in the question is 3 hours, substitute 3 for h and solve.

- C. How long will it take them to get to Bakersville?

Answer: $257 = 60h + 57 \rightarrow 200 = 60h \rightarrow 3\frac{1}{3} \text{ hours} = h$

The given information in the question is 257 miles to Bakersville, substitute 257 for d and solve.

Practice Problems for Topic XII:

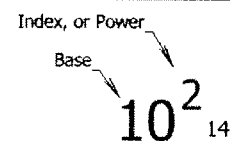
Problem Situation: Georgette goes on a diet. She currently weighs 220 lbs and estimates she can lose 2.5 lbs. a week. Her goal weight is 130 lbs.

- a) Write an equation to model Georgette's weight, p , in terms of time, w .
- b) How much will she weigh in 12 weeks?
- c) How long will it take her to reach her goal weight?

Answers: a) $p = -2.5w + 220$ b) $p = -2.5(12) + 220 = 190 \text{ lbs.}$ c) $130 = -2.5w + 220; w = 36 \text{ weeks}$

XIII: Exponents and Their Properties

Parts of Exponents: the large number is the base, the small number is the index, power or exponent.



Hint: Don't forget to follow the order of operations when simplifying

Properties of Exponents

Multiplication: $(x^m)(x^n) = x^{(m+n)}$	Add exponents of terms with like bases
Division: $\frac{x^m}{x^n} = x^{(m-n)}$	Divide exponents of terms with like bases
Powers: $(x^m)^n = x^{(m \cdot n)}$	Multiply exponents
Zero Power: $x^0 = 1; x \neq 0$	Anything to the zero power is 1
Negative Exponents: $x^{-m} = \frac{1}{x^m}$	If a term has a negative exponent, change it's position in the fraction and the exponent becomes a positive
Fractions: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$	Apply the exponent to the numerator and denominator of a fraction

Examples for Topic XIII: Simplify.

A. $(3x^4y^2)(4xy^5)$	Recognize that the operation is multiplication
$12(x^4y^2)(xy^5)$	Multiply the coefficients
$12x^5y^7$	Add exponents of like bases.

B. $(-2a^3bc^4)^3$	Recognize that this is the power rule
$-8(a^3bc^4)^3$	Raise -2 to the 3 rd power $(-2)^3 = -8$
$-8a^9b^3c^{12}$	Multiply exponents

C. $\frac{42m^5f}{-3m^3f^2}$	Recognize that the operation is division
$-14\frac{m^5f}{m^3f^2}$	Divide the coefficients
$-14\frac{m^2}{f}$ or $\frac{-14m^2}{f}$	Subtract the exponents of like bases.

D. $5x^0y^{-4}$	Recognize that this is the zero power rule and there is a negative exponent
$5(1)y^{-4}$	Simplify the term with the zero exponent to 1
$5y^{-4}$	Multiply the constants
$\frac{5}{y^4}$	Move the negative term to the other side of the fraction bar and make the exponent positive

Practice Problems for Topic XIII: Simplify.

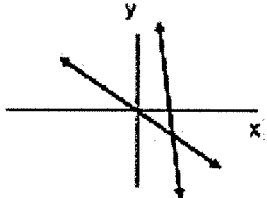
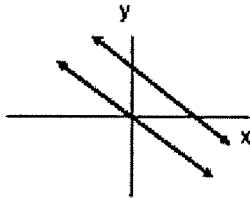
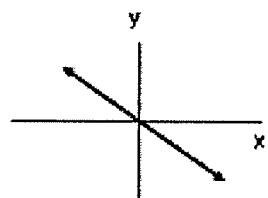
a) $\frac{6x^7y^5}{3x^{-2}}$ b) $\left(\frac{3x^2}{2}\right)^3$ c) $(3a^2)(4a^6b)$ d) $(8h^5j^3k)^0$ e) $(x^5)(2x)^3$

Answers: a) $2x^9y^5$ b) $\frac{27x^6}{8}$ c) $12a^8b$ d) 1 e) $(x^5)(8x^3) = 8x^8$

XIV: Systems of Equations

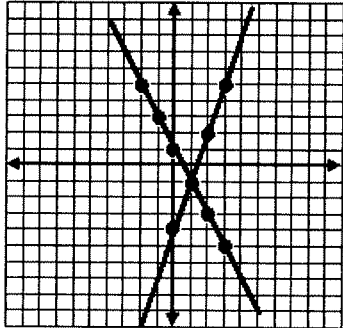
A system of equations is a set of two or more equations that use the same set of variables. Systems can be solved by graphing or using the algebraic methods of substitution or elimination. Systems can have one solution, no solution or many solutions. The point or points shared by the equations is the solution.

Types of Solutions:

Type of Solution	One Solution	No Solution	Many Solutions
Graph			
Examples of Solutions	(x, y)	(No solution)	(All real numbers)

Examples for Topic XIV:

a. Solve the system by graphing

<p>A. $\begin{cases} y = 3x - 4 \\ y = -2x + 1 \end{cases}$</p> <ul style="list-style-type: none"> Graph the first line ($y = 3x - 4$) Graph the second line ($y = -2x + 1$) The solution is the point where the lines intersect Answer: $(1, -1)$ 	
---	--

b. Solve the system by substitution

<p>B. $\begin{cases} y = 3x - 4 \\ y = -2x + 1 \end{cases}$</p> <table border="1" style="width: 100%;"> <tr> <td>$3x - 4 = -2x + 1$</td> <td>Since both equations equal y, substitute one for y.</td> </tr> <tr> <td>$x = 1$</td> <td>Solve for x.</td> </tr> <tr> <td>$y = 3(1) - 4 = -1$</td> <td>To find y, substitute 1 into one of the equations for x.</td> </tr> <tr> <td>$(1, -1)$</td> <td>Write the answer as a point</td> </tr> </table>	$3x - 4 = -2x + 1$	Since both equations equal y , substitute one for y .	$x = 1$	Solve for x .	$y = 3(1) - 4 = -1$	To find y , substitute 1 into one of the equations for x .	$(1, -1)$	Write the answer as a point	<p>C. $\begin{cases} y = 5x - 1 \\ -2x + 3y = 23 \end{cases}$</p> <table border="1" style="width: 100%;"> <tr> <td>$-2x + 3(5x - 1) = 23$</td> <td>Substitute the y in the second equation with $5x - 1$</td> </tr> <tr> <td>$x = 2$</td> <td>Solve for x.</td> </tr> <tr> <td>$y = 5(2) - 1 = 9$</td> <td>To find y, substitute 2 into one of the equations for x.</td> </tr> <tr> <td>$(2, 9)$</td> <td>Write the answer as a point</td> </tr> </table>	$-2x + 3(5x - 1) = 23$	Substitute the y in the second equation with $5x - 1$	$x = 2$	Solve for x .	$y = 5(2) - 1 = 9$	To find y , substitute 2 into one of the equations for x .	$(2, 9)$	Write the answer as a point
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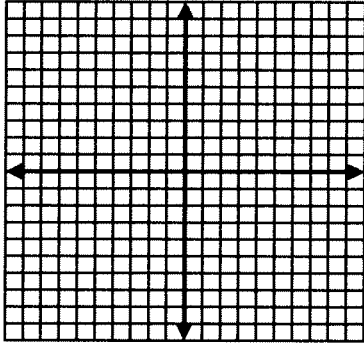
c. Solve the system by elimination

<p>D. $\begin{cases} 4x - 2y = 7 \\ x + 2y = 3 \end{cases}$</p> <table border="1" style="width: 100%;"> <tr> <td>$5x = 10$</td> <td>Add the equations together to eliminate y terms.</td> </tr> <tr> <td>$x = 2$</td> <td>Solve for x.</td> </tr> <tr> <td>$2 + 2y = 3$ $y = 0.5$</td> <td>To find y, substitute 2 into one of the equations for x and solve.</td> </tr> <tr> <td>$(2, 0.5)$</td> <td>Write the answer as a point</td> </tr> </table>	$5x = 10$	Add the equations together to eliminate y terms.	$x = 2$	Solve for x .	$2 + 2y = 3$ $y = 0.5$	To find y , substitute 2 into one of the equations for x and solve.	$(2, 0.5)$	Write the answer as a point	<p>E. $\begin{cases} 3x + 7y = 15 \\ 5x + 2y = -4 \end{cases}$</p> <table border="1" style="width: 100%;"> <tr> <td>$\begin{cases} -15x - 35y = -75 \\ 15x + 6y = -12 \end{cases}$</td> <td>No variables will be eliminated by addition alone. Multiply the 1st equation by -5 and the 2nd equation by 3.</td> </tr> <tr> <td>$-29y = -87$ $y = 3$</td> <td>Add the equations together to eliminate x term. Solve for y.</td> </tr> <tr> <td>$3x + 7(3) = 15$ $x = -2$</td> <td>To find x, substitute 3 into one of the equations for y and solve.</td> </tr> <tr> <td>$(-2, 3)$</td> <td>Write the answer as a point</td> </tr> </table>	$\begin{cases} -15x - 35y = -75 \\ 15x + 6y = -12 \end{cases}$	No variables will be eliminated by addition alone. Multiply the 1 st equation by -5 and the 2 nd equation by 3.	$-29y = -87$ $y = 3$	Add the equations together to eliminate x term. Solve for y .	$3x + 7(3) = 15$ $x = -2$	To find x , substitute 3 into one of the equations for y and solve.	$(-2, 3)$	Write the answer as a point
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$(-2, 3)$	Write the answer as a point																

Practice Problems for Topic XIV:
Solve the system using the indicated method.

a) Solve by graphing.

$$\begin{cases} y = 4x + 2 \\ y = 2x + 4 \end{cases}$$



Answer: _____

b) Solve by substitution

$$\begin{cases} y = 9x - 5 \\ y = -6x + 25 \end{cases}$$

Answer: _____

c) Solve by substitution

$$\begin{cases} y = -x + 7 \\ 2x + 3y = 14 \end{cases}$$

Answer: _____

d) Solve by elimination.

$$\begin{cases} 5x - 6y = -32 \\ 3x + 6y = 48 \end{cases}$$

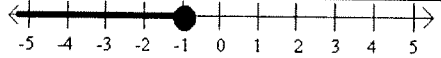
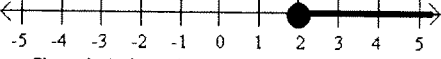
Answer: _____

Answers: a) (1, 6) b) (2, 13) c) (7, 0) d) (2, 7)

XV: Solving Linear Inequalities

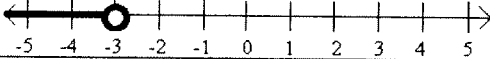
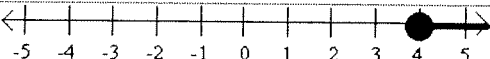
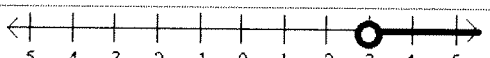
Solving linear inequalities is similar to solving linear equations. However, if you multiply or divide by a negative, you must change the inequality sign. Unlike a linear equation, the answer is graphed on a number line.

Name	Symbol	Example	Graph
Less Than	<	$x < 4$ "x is less than 4"	<p>Open circle indicates we do not include the 4 Highlight all numbers less than 4</p>
Greater Than	>	$x > -4$	

		"x is greater than -4"	Open circle indicates we do not include the -4 Highlight all numbers greater than -4
Less Than or Equal To	\leq	$x \leq -1$ "x is less than or equal to -1"	 Closed circle indicates we include the -1 Highlight all numbers less than -1
Greater Than or Equal To	\geq	$x \geq 2$ "x is greater than or equal to 2"	 Closed circle indicates we include the 2 Highlight all numbers greater than -4

Examples for Topic XV:

Solve the inequality and graph the solution.

A. $x+3 < 0$	Subtract 3 from both sides of the inequality
$x < -3$	Solution: "x is less than -3"
	Graph the solution using an open circle because the sign is "less than" and does not include the -3, only everything <i>less than</i> -3.
B. $x-4 \geq 0$	Add 4 to both sides of the inequality
$x \geq 4$	Solution: "x is greater than or equal to 4"
	Graph the solution using a closed circle because the sign is "greater than or equal to" and includes the 4 and everything greater than 4.
C. $-2x+5 < -1$	Subtract 5 from both sides of the inequality
$-2x < -6$	Divide both sides by -2. **Change the direction of sign because you divided by a negative**
$x > 3$	Solution: "x is greater than 3"
	Graph the solution using an open circle because the sign is "greater than" and does not include the 3, only everything <i>greater than</i> 3.

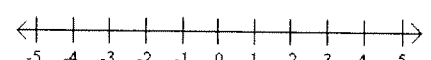
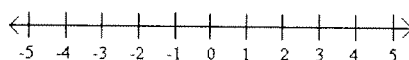
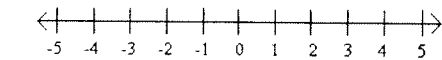
Practice Problems for Topic XV:

Solve the inequality and graph the solution.

a) $-3x < -12$

b) $3x - 12 \geq -6$

c) $-\frac{x}{2} \geq 1$



Answers:

a) $x > 4$

b) $x \geq 2$

c) $x \leq -2$

